

MEASURING THE MASS OF 4U0900-40 DYNAMICALLY

J. F. Dolan

Exploration of the Universe Division,
NASA Goddard Space Flight Center, Greenbelt, MD 20771;

Joseph.F.Dolan@nasa.gov

Paul B. Etzel

Department of Astronomy,
San Diego State University, San Diego, CA 92182-1221;

etzel@algol.sdsu.edu

Patricia T. Boyd

University of Maryland Baltimore County;
Exploration of the Universe Division,
NASA Goddard Space Flight Center, Greenbelt, MD 20771;

padi@milkyway.gsfc.nasa.gov

ABSTRACT

Accurate measurements of neutron star masses are needed to constrain the equation of state of neutron star matter - of importance to both particle physics and the astrophysics of neutron stars - and to identify the evolutionary track of the progenitor stars that form neutron stars. The best measured values of the mass of 4U0900-40 (= Vela XR-1), 1.86 ± 0.16 Msun (Barziv et al. 2001) and 1.93 ± 0.20 Msun (Abubekrov et al. 2004), make it a leading candidate for the most massive neutron star known. The direct relationship between the maximum mass of neutron stars and the equation of state of ultra-dense matter makes 4U0900-40 an important neutron star mass to determine accurately. The confidence interval on previous mass estimates, obtained from observations that include parameters determined by non-dynamical methods, are not small enough to significantly restrict possible equations of state. We describe here a purely dynamical method for determining the mass of 4U0900-40, an X-ray pulsar, using the reprocessed UV pulses emitted by its B0.5Ib companion. One can derive the instantaneous radial velocity of each component by simultaneous X-ray and UV observations at the two quadratures of the system. The Doppler shift caused by the primary's rotational velocity and the illumination pattern of the X-rays on the primary, two of the three principal contributors to the uncertainty on the derived mass of the neutron star, almost exactly cancel by symmetry in this method. A heuristic measurement of the mass of 4U0900-40 using observations obtained previously with the High Speed Photometer on HST is given in Appendix A.

Keywords: -- methods: analytical -- binaries: close -- stars: neutron -- X-rays:
individual: 4U0900-40

1. INTRODUCTION

Knowing the properties of matter at densities greater than that found in nuclei of atoms is necessary in order to understand the physics of neutron stars and the Big Bang (Stock 1989). The equation of state of matter at high density, relating pressure to energy density, is usually formulated in terms of a compression parameter, K . If $K > 300$ MeV, the matter is relatively incompressible and the equation of state is called “stiff”; if $K < 150$ MeV, it is called “soft”. The maximum mass of a neutron star is directly related to the equation of state, and an accurate value of this maximum mass would significantly constrain the value of K (Glendenning 1986). For a soft equation of state, the maximum is near $1.5 M_{\text{sun}}$, increasing to $\sim 2 M_{\text{sun}}$ for a stiff value of K (Barziv et al. 2001 and references therein). The uncertainty in the value of K is the principal component of the uncertainty in neutron star models (Shapiro & Teukolsky 1983; Arnett & Bowers 1977). Determining the maximum mass of neutron stars would also assist in identifying the evolutionary track of their progenitor stars, a currently open question in the field of stellar evolution (Brown & Bethe 1994; Datta 1988).

4U0900-40, J0751+1807, and 4U1700-37 are the three neutron stars with the largest known masses (Lattimer & Prakash 2004). The best estimates for the mass of 4U0900-40 are $1.93 \pm 0.20 M_{\text{sun}}$ (Abubkerov et al 2004) and $1.86 \pm 0.16 M_{\text{sun}}$ (Barziv et al. 2001). Abubkerov et al. state that their quoted error demarcates the 95% confidence interval on the mass, but add that it “cannot be considered very trustworthy” for reasons we discuss below. The error quoted by Barziv et al. is a 1σ confidence interval. Essentially all published measurements of 4U0900-40's mass give $m > 1.8 M_{\text{sun}}$, although with larger confidence intervals than those above. Nice et al. (2004) give a 95% confidence interval on the mass of the radio pulsar J0751+1807 of $(1.6 - 2.8) M_{\text{sun}}$ based on pulse timing measurements fitted to orbital decay caused by the emission of gravitational radiation. Clark et al.

(2002) estimate the mass of 4U1700-37 to be $2.44 \pm 0.27 M_{\text{sun}}$ based on the spectroscopic mass function of the 4U1700-37/HD153919 system and a mass of the O6.5 Iaf star derived from a non-LTE analysis of its spectrum. Clark et al. note that if a minimum mass of $50 M_{\text{sun}}$ is adopted for HD153919, the mass of 4U1700-37 can be as low as $1.83 M_{\text{sun}}$, although they consider this possibility unlikely based on Monte Carlo simulations. Heap & Corcoran (1992) derived a mass of $1.8 \pm 0.4 M_{\text{sun}}$ for 4U1700-37 using the assumption that the radius of the primary was appropriate to its spectral type. Lattimer & Prakash (2004) note that 4U1700-37 may be a black hole, not a neutron star, based on, among other things, the lack of X-ray pulses from the object (Dolan et al. 1980). On balance, we consider 4U0900-40 to be the leading candidate for the neutron star with the maximum known mass.

1.1. The 4U0900-40 System

4U0900-40 is in an eclipsing binary system with a B0.5Ib primary, HD77581. The orbital period of the system is 8.96 d (van Kerkwijk et al 1995b). The neutron star is an X-ray pulsar with a rotation period of 283 s (Bildsten et al. 1997). The orbit is elliptical, $e = 0.0898 \pm 0.0012$ (Bildsten et al. 1997) and pulse-timing observations give $[a(x)/c] \sin i = 113.89 \pm 0.13$ s (Bildsten et al. 1997; Deeter et al. 1987). The inclination of the system is $i = 76^\circ (+5^\circ, -9^\circ)$ (Dolan & Tapia 1988).

1.2. Current Status of Mass Determinations of 4U0900-40

Measurements of the mass of some radio pulsars can be made with associated uncertainties $< 0.1 M_{\text{sun}}$ (Thorsett & Chakrabarty 1999; Thorsett et al. 1993) because they are derived using a dynamical method. If the instantaneous center of mass (COM) velocity of both stars in a binary system can be directly observed, then $v(1)/v(2) = v_r(1)/v_r(2) = m(2)/m(1)$. For massive X-ray binaries, the radial velocity of the neutron star relative to the COM of the system can be directly determined from pulse timing observations if the neutron star is an X-ray pulsar.

The radial velocity of the primary relative to the COM of the system is more difficult to infer from observations. The radial velocity curve of HD77581 is confused by “systematic distortions of the amplitude” (Bahcall 1978), caused by the primary's stellar wind outflow and the heating of the hemisphere facing the neutron star by X-rays, and “the presence of pulsation-like perturbations” (van Kerkwijk et al. 1995a) uncorrelated with orbital phase, perhaps excited by tidal forces from the neutron star and complicated by systematic effects on the spectra caused by a photo-ionization wake trailing the neutron star in its orbit (Kaper et al. 1994). “Tidal distortion . . . causes systematic deviations of the radial velocity curve, since for a deformed star 'center-of-light' radial velocity measurements do not necessarily reflect the actual center of mass velocity” (Barziv et al 2001). The systematic errors in the amplitude of the radial velocity curve caused by these effects contribute an irreducible uncertainty to the derived radial velocity of the primary relative to the COM that propagates through to the confidence interval on the neutron star's mass.

Barziv et al. (2001) note that “the individual radial-velocity measurements (of HD77581) deviate significantly from the best-fit Keplerian orbit.” Abubkerov et al. (2004) state that none of their orbital solutions acceptably represent the observations under the assumptions of the χ^2 test using a Roche lobe model, even when the standard deviations of the individual data points were arbitrarily increased by a factor of 1.5.

2. REPROCESSED UV PULSES FROM HD77581

Basko and Sunyaev (1973) and Avni and Bahcall (1974) predicted that X-ray pulses illuminating the facing hemisphere of a companion star should be absorbed into its photosphere and increase the ionization state of several abundant species of metal ions. When the X-ray beam has passed, the temperature inversion layer in the photosphere quickly reradiates the excess heat, primarily in the recombination lines of these metals (Dahab 1974; Alme & Wilson 1974). The reprocessed UV radiation that results is pulsed with a period near that of the X-ray pulses.

Reprocessed UV radiation has been observed from HD77581 in an emission line located in the absorption trough of the P Cygni profile of the Si IV resonance line at 1402 Å (Boroson et al. 1996; Dolan et al. 1998). Boroson et al. also detected reprocessed UV pulses in the N V recombination line at 1238/42 Å. The pulse profile of the Si IV emission resembles that observed simultaneously in 1.2 - 2.3 keV X-rays (Boroson et al. 1996).

The pulsed fraction observed by Dolan et al. (1998) at binary phase $\psi = 0.92$ in a 200 Å wide bandpass that included the Si IV line was $1.2 \pm 0.6 \%$. (The center of X-ray eclipse is defined as $\psi = 0$.) Only 6% of the visible hemisphere of HD77581 is illuminated by X-ray pulses at $\psi = 0.92$. Boroson et al. (1996) observed a pulsed amplitude of 3% in the 1402 Å line at $\psi = 0.46$. Reprocessed UV pulses should be visible from HD77581 in the Si IV line at nearly every phase outside of X-ray eclipse.

Observations of reprocessed optical radiation from X-ray binary systems with low and intermediate mass stellar components locate the origin of reprocessed emission lines in the high photosphere of the companion star. The C III/N III blend at 4640 - 50 Å caused by the Bowen fluorescence mechanism of He II Ly α

arises in the photosphere of the companion star's facing hemisphere in Sco XR-1, not from the accretion flow or a hot spot on the accretion disk (Steehgs & Casares 2002; Schachter et al. 1989). The majority of reprocessed line emission in Her X-1 originates on or near the mass-losing primary (Still et al. 1997). In GX339-4, the reprocessed NIII emission lines are “most plausibly” formed by irradiation of the companion star and their velocities indicate the companion star's orbital motion (Hynes et al. 2003). We expect the site from which the reprocessed radiation arises in the 4U0900-40 system to move with the primary.

Only one X-ray pulse from 4U0900-40 illuminates HD77581 at any one time because the light travel time across the radius of the primary is $\sim 0.2 p(x)$, where $p(x)$ is the 283 s X-ray pulse period. The frequency of the reprocessed UV pulses is Doppler shifted by the orbital motion of the primary if they originate at a site attached to HD77581. We show below how in this case simultaneous measurements of the X-ray pulse period and period of the reprocessed UV pulses at quadrature can be analyzed to determine the instantaneous radial velocity of each component.

3. DYNAMICAL METHODS OF MASS DETERMINATION

3.1. Standard Method

A binary star system is a close approximation to the Newtonian two-body problem. The ratio of the masses, m , of the stars are related to their instantaneous velocities, v , and semi-major axes, a , relative to the COM by

$$m(x)/m(o) = v(o)/v(x) = v_r(o)/v_r(x) = a(o)/a(x) \quad [1]$$

where v_r is the radial velocity of the star relative to the COM. In the system of units in which $4\pi^2/G = 1$ (mass in solar masses, distance in AU, time in years),

$$P^2 = a^3/[m(o) + m(x)] \quad [2]$$

where P is the period of the binary orbit and $a = a(o) + a(x)$.

In theory, knowing P , $v_r(o)/v_r(x)$ and $a(x)$ simultaneously allows one to determine $m(x)$ and $m(o)$ from [1] and [2]. In practice, several complications arise. P is well determined, but X-ray pulse-timing observations give $a(x) \sin i$, not $a(x)$ alone (Deeter et al. 1987). The inclination, i , can be determined from polarimetric observations of scattered light in the system (Dolan & Tapia 1988) or by assuming the shape of the primary and modeling its photometric light curve (Bahcall 1978) or the length of the X-ray eclipse (Heap & Corcoran 1992; van Kerkwijk et al. 1995b). Other complications that contribute significantly to the uncertainty on the derived mass are more difficult to address.

The standard dynamical method for determining the mass of 4U0900-40 using reprocessed UV pulses from HD77581 corrects the velocity corresponding to the Doppler shift of the observed pulse period to the velocity of the COM of the

primary. The correction involves the rotational velocity of the primary and the shape of the primary's surface illuminated by X-rays that is visible from Earth.

The shape of the surface area on the visible hemisphere of the primary emitting reprocessed UV pulses varies as a function of orbital phase. The observed pulse period that results is Doppler shifted by an amount corresponding to the intensity weighted radial velocity, v'_r , of the center of light (COL) of the reprocessed photons,

$$v'_r = \int v_r(\theta, \varphi) I(\theta, \varphi) d\Omega / \int I(\theta, \varphi) d\Omega \quad [3]$$

where I is the intensity of each element of surface area emitting reprocessed UV pulses at position (θ, φ) in spherical co-ordinates, and the integrals are over the surface area of the visible hemisphere of the primary that is illuminated by X-rays. $v_r(\theta, \varphi)$ is the radial velocity of each element of surface area, which is the projected velocity on the line of sight of the vector sum of the COM velocity of the star, $v(o)$, and the element's rotational velocity around the polar axis of the star, $v(r)$. To correct the velocity corresponding to the observed pulse period to the COM of the primary we need to know both the rotational velocity of the primary and the intensity distribution of reprocessed pulses on its visible hemisphere.

In Appendix A, we use the standard dynamical method to derive the mass of 4U0900-40 based on our observations of reprocessed UV pulses on 1993 January 6 (Dolan et al. 1998). The confidence interval on the mass derived is so large that the result is of little use. The three principal contributions to the uncertainty in the derived mass come from the uncertainties in the rotational velocity of HD77581 (Zuiderwijk 1995), the model of the reprocessed light distribution, and the inclination of the system.

3.2 Quadrature Method

The mass of 4U0900-40 can be measured dynamically using reprocessed UV pulse timing observations without requiring a knowledge of either the rotational velocity of the primary or the pattern of its reprocessed light distribution. Consider the system at its two quadratures, which we denote QI and QII. Quadratures occur when the line of centers of the two stars is perpendicular to the line of sight to the COM of the binary, i. e., when the neutron star crosses the line of nodes of its orbit. (The line of nodes is the intersection of the orbital plane with the plane of the sky.) Without loss of generality, we will assume direct rotation of the primary and assign the approaching primary to QI (Fig. 1).

The radial velocities corresponding to the observed Doppler shift of the reprocessed UV pulses at QI and QII relative to the COM of the system are

$$v_r(\text{I}) = -|v_r(o)| + |v'_r| ; \quad v_r(\text{II}) = |v_r(o)| - |v'_r| \quad [4]$$

where $v_r(o)$ is the radial velocity of the COM of the primary relative to the COM of the binary, and v'_r is the radial velocity representing the contribution to the Doppler shift of the rotational velocity of the primary as convolved through the intensity distribution of the reprocessed pulses from its visible hemisphere. If the orbit of 4U0900-40 were circular, then by symmetry $|v'_r|$ would be the same at the two quadratures. The orbit is actually elliptical, with $e = 0.0898 \pm 0.0012$ and $\omega = 152.6^\circ \pm 0.9^\circ$ (Bildsten et al. 1997; Deeter et al. 1987; Boynton et al. 1986). The separation between the two components is slightly different at the two quadratures. The effect of this on the radial velocity contribution from the rotational velocity of the primary as convolved through the COL of the reprocessed UV pulses is almost negligible; its magnitude is calculated in Appendix B. We shall take $|v'_r|$ to be the same at both quadratures here.

Because the orbit of HD77581 has the same ellipticity as that measured for the X-ray source, $v_r(o)$ is different at QI and QII. Let $\langle v \sin i \rangle$ be the mean velocity of HD77581 in its projected orbit. Set $|v_r(o)| = \beta \langle v \sin i \rangle$ at QI and $|v_r(o)| = \alpha \langle v \sin i \rangle$ at QII, where α and β are positive numbers. Adding Eqs. [4] together then gives

$$v_r(I) + v_r(II) = (\alpha - \beta) \langle v \sin i \rangle \quad [5]$$

So

$$\langle v \sin i \rangle = [v_r(I) + v_r(II)]/(\alpha - \beta) \quad [6]$$

The COM velocity of the primary at second quadrature is then

$$v_r(o)|_{II} = \alpha \langle v \sin i \rangle = [\alpha/(\alpha - \beta)] [v_r(I) + v_r(II)] \quad [7]$$

The ratio $\alpha/(\alpha - \beta)$ can be calculated from the orbit of 4U0900-40 determined from X-ray observations (Bildsten et al. 1997; Deeter et al. 1987); its value is 13.1 ± 0.2 . ^(Appendix C)
This ratio necessarily has the same value for the orbit of the primary.

Eq. [7] therefore gives the velocity of the primary at second quadrature without the uncertainties introduced from measurements of its optical velocity curve.

Knowing $v_r(x)$ at QII from simultaneous X-ray observations gives us the ratio of the velocities of the two components. The solution for the masses then proceeds as in the standard dynamical method, but without any contribution to the uncertainty in the mass of 4U0900-40 from either the rotational velocity of the primary or the pattern illuminated by X-rays on its visible hemisphere.

4. DISCUSSION

Converting the observed pulse period of the reprocessed UV pulses into the corresponding Doppler velocity of the primary requires knowing the period of the X-ray pulses from 4U0900-40 seen by HD77581. For a circular orbit, the two components always move perpendicular to the line joining them, and the X-ray pulse period at the COM of the orbit of 4U0900-40 as observed at Earth would be the pulse period of the X-rays observed at HD77581. For an elliptical orbit, 4U0900-40 has a non-zero radial velocity relative to HD77581 at every binary phase except periastron and apoastron. This radial velocity is known from the X-ray orbit. The X-ray pulse period observed at HD77581 is the period at the COM of the orbit as Doppler shifted by this radial velocity. The velocity of the primary is then derived from the Doppler shift between this incident X-ray period and the observed period of the reprocessed UV pulses.

The Doppler shift appearing in Eq. [5] can be estimated from the X-ray orbit (Bildsten et al. 1997). $(\alpha - \beta) \langle v \sin i \rangle = 22 \text{ km s}^{-1}$ for 4U0900-40. For $m(o)/m(x) = 10$, a reasonable estimate, $(\alpha - \beta) \langle v \sin i \rangle = 2.2 \text{ km s}^{-1} = v_r(I) + v_r(II)$ for HD77581. The corresponding Doppler shift in the 283 s pulse period is 2.1 ms.

To significantly constrain the compression parameter K in the equation of state of ultra-dense matter, the confidence interval on the maximum mass of a neutron star needs to be restricted to $\sim \pm 0.1 M_{\text{sun}}$. Based on the error analysis in Appendix A, this would require measuring the period of the reprocessed pulses to $\sim \pm 0.3 \text{ ms}$ when using the quadrature method. Measuring the 283 s reprocessed pulse period with this precision seems feasible (cf. Dolan et al. 1998).

ACKNOWLEDGEMENTS. We thank R. J. Drachman for assistance in using Mathematica to calculate the radial velocity contribution from the rotational velocity of HD77581. We also thank an anonymous referee for a careful reading of the paper and many helpful suggestions for improving its presentation.

APPENDIX A

A HEURISTIC DERIVATION OF THE MASS OF 4U0900-40

We observed HD77581 on 1993 January 6 (Dolan et al 1998) using the High Speed Photometer on the Hubble Space Telescope. The observation started on JD 244 8983.7815, at orbital phase 0.9392 ± 0.0001 using the orbital ephemeris of Bildsten et al (1997). We detected UV pulses during the first 600 s of the observation, a duration of 0.0008 in phase, in a 200 Å wide bandpass that included the Si IV resonance line at 1403 Å.

To illustrate how the uncertainty in the rotational period of the primary and the X-ray illumination pattern on its surface contribute to the uncertainty on the value of the mass of the neutron star derived using the standard dynamical method, let us assume for simplicity that the binary system has a circular orbit. Then the velocity of the neutron star is always tangential to its radius vector from HD77581, and the X-ray pulses arriving at the surface of the primary always have the period $p(x)$, where $p(x)$ is the X-ray pulse period at the COM of the orbit. ($p(x)$ is the value of the pulse period given in the X-ray ephemerides.) Any velocity of the COM of the binary system relative to the solar system does not affect $v(o)/v(x)$ in Eq. [1], the ratio of the velocity of the components relative to the COM of the binary. The velocity of the 4U0900-40 at every point in its projected orbit is then $v(x) = 2\pi a(x)/P = 277.0 \pm 0.3 \text{ km s}^{-1}$.

We will assume that the X-ray pulses uniformly illuminate the facing hemisphere of HD77581 when averaged over time. (Although the illumination pattern can be modeled based on the parameters of the system, in practice it contributes an unknown systematic error to the derived mass of the neutron star when using the standard dynamical method.) Each illuminated element of the primary's surface will see the period of the illuminating X-ray pulses Doppler shifted by an amount

corresponding to that element's rotational velocity around the polar axis of HD77581 projected onto the line of sight from the element to the neutron star. The rotational velocity of HD77581 at its equator is $v(\text{eq}) = 116 \pm 6 \text{ km s}^{-1}$ based on an analysis of selected absorption line profiles in its spectrum (Zuiderwijk 1995). We assume that the polar axis of HD77581 is perpendicular to its orbital plane. At $\psi = 0.9396 \pm 0.0004$, the 600 s long phase interval during which pulsations were detected, the component of the rotational velocity radially directed toward the neutron star at the COL of the illuminated lune seen from Earth is

$$v_r'' = (0.4715 \pm 0.0001) v(\text{eq}) = 55 \pm 3 \text{ km s}^{-1} \quad [\text{A1}]$$

At the epoch of our observations, the ephemeris of Bildsten et al. (1997) gives $p(x) = 283.356 \pm 0.004 \text{ s}$ and that of Inam & Baykal (2000) gives $p(x) = 283.341 \pm 0.008 \text{ s}$. We adopt the weighted average, $p(x) = 283.351 \pm 0.004 \text{ s}$. The X-ray pulse period observed at the COL of the illuminated lune is $p'' = p(x)[1 - (v_r''/c)] = 283.304 \pm 0.005 \text{ s}$.

The reprocessed UV pulses are emitted at the COL of the illuminated lune with period p'' . The component of the rotational velocity of the COL of the reprocessed pulses radially directed away from Earth, is

$$v_r' = (0.294 \pm 0.011) v(\text{eq}) = 34.1 \pm 2.2 \text{ km s}^{-1} \quad [\text{A2}]$$

The confidence interval associated with v_r' is larger than that associated with v_r'' because the uncertainty in the inclination of the system contributes only to v_r' . (The location of the rotational pole of HD77581 on its visible hemisphere is a function both of its inclination and the orbital phase.) The reprocessed pulses from the COL are emitted with period $p' = p''[1 + (v_r'/c)] = 283.331 \pm 0.005 \text{ s}$ as seen from

Earth.

The difference between $p(o)$, the period of the reprocessed pulses observed at Earth, and p' is caused by the radial velocity of the COM of the primary, $v_r(o)$. The heliocentric period of the UV pulses we observed was 283.320 ± 0.045 s; the large uncertainty in $p(o)$ is caused by the short duration over which pulses were observed, only two pulse periods long. To more clearly illustrate the contribution of the uncertainty in the rotational velocity of the primary to the uncertainty in the derived mass, we shall treat $p(o)$ as having negligible uncertainty here. Then

$$v_r(o) = [p(o) - p']c/p' = 11.6 \pm 5.3 \text{ km s}^{-1} \quad [\text{A3}]$$

At $\psi = 0.9396$, $v_r(x) = 102.6 \pm 0.7 \text{ km s}^{-1}$ for a circular orbit, so $v_r(o)/v_r(x) = 0.113 \pm 0.052 = m(x)/m(o)$.

The uncertainty in the rotational velocity of the primary contributes about one-third of the uncertainty in the mass ratio of the components. The remainder is contributed primarily by the uncertainty on $p(x)$. $p(x)$ was extrapolated from X-ray ephemerides in this calculation; the uncertainty on $p(x)$ can be decreased by continuous observations of X-ray pulse arrival times during the times at which one observes the reprocessed UV pulses. The uncertainty on the illumination pattern of the X-ray pulses on the primary also contributes an unknown systematic error to the result.

The sum of the masses, from Eq. [2], is $m(o) + m(x) = (19.74 \pm 0.07) M_{\text{sun}}/\sin^3 i = 21.7 (+3.3, -1.2) M_{\text{sun}}$. The non-symmetric confidence interval on the sum of the masses is caused by the non-symmetric confidence interval on $\sin i$, the principal contributor to the uncertainty. A better estimate of the inclination, perhaps from simultaneous polarimetric and photometric observations, is necessary to improve

this uncertainty .

The simultaneous equations for the sum and ratio of the masses give $m(x) = 2.2 (+1.0, -0.9) M_{\text{sun}}$. We emphasize that this calculation is intended to illustrate the size of the errors present in the standard dynamical method, and that the the mass of 4U0900-40 we derive here is of heuristic value only. The quadrature method achieves superior accuracy because it need to measure only $p(o)$ and $p(x)$.

APPENDIX B

SYMMETRY OF v_r AT THE TWO QUADRATURES

We can calculate the weighted radial velocity contribution caused by the rotation of HD77581 at the two quadratures using Eq. [3]. We will assume a simplified model in which the star is a sphere, its rotational axis is perpendicular to the line of sight, and the X-rays uniformly illuminate the surface visible from 4U0900-40.

We assume that the intensity of the UV pulses, $I(\theta, \phi)$ in Eq. [3], is proportional to the X-ray heating at each element of the illuminated stellar surface, and that this is proportional to the inverse square of the distance of the element from 4U0900-40. The radial velocity of each surface element, v_r in Eq. [3], is now only that caused by the rotation of the star.

The separation of the centers of the two components is 3.55×10^{12} cm at first quadrature (QI) and 4.15×10^{12} cm at second quadrature (QII). The surface illuminated at QI is also illuminated at QII. An additional circular band at the edge of this illuminated area is also illuminated at QII. This additional illuminated band is close to the rotational axis of the primary, i.e., near the $v_r = 0$ line on the hemisphere visible from Earth. Taking $30.0 R_{\text{sun}}$ as the radius of HD77581 (van Kerkwijk et al. 1995b) and 116 km s^{-1} as the rotational velocity at its equator (Zuiderwijk 1995), we find the radial velocity contribution caused by stellar rotation is $|v_r| = 48.6 \text{ km s}^{-1}$ at QI and 48.1 km s^{-1} at QII. As expected from the lower radial velocity of the additional area illuminated at QII, $|v_r|$ is slightly smaller at QII than at QI.

The difference between the value of $|v_r|$ at the two quadratures must be smaller than this calculated value of 0.5 km s^{-1} . HD77581 is not spherical, but tidally deformed with a tear-drop shape pointed at 4U0900-40. The sub-secondary point is closer to the X-ray pulsar than it is in the spherical approximation assumed in

the calculation, and so $I(\theta, \varphi)$ in the surface area illuminated at both QI and QII is brighter relative to the intensity from the additional circular band illuminated at QII. This closer approach to symmetry brings the value of $|v'_r|$ at QII closer to that at QI than that calculated for a spherical star. The $i = 76^\circ$ inclination of the orbit also tilts the rotational axis away from the perpendicular to the line of sight. This lowers the value of $|v'_r|$ from that calculated in the spherical case by a similar amount at both quadratures. A fixed fractional difference between the two values then translates to a lower value of the difference in units of km s^{-1} . We estimate that the two effects lower the difference between the values of $|v'_r|$ by at least a factor of 2 from the spherical case in the 4U0900-40 system, to $\sim 0.25 \text{ km s}^{-1}$.

A residual of 0.25 km s^{-1} from imperfect cancellation of stellar rotation at the two quadratures contributes 0.2 ms to the Doppler shift of the UV pulses in Eq. [7]. This is within the $\pm 0.3 \text{ ms}$ measurement error estimated in Section 4.

APPENDIX C

THE RATIO $\alpha/(\alpha - \beta)$

We use the ephemeris of Bildsten et al. (1997), with orbital period $P = 8.964368 \pm 0.000040$ d, longitude of periastron $\omega = 152.6^\circ \pm 0.9^\circ$, and eccentricity $e = 0.0898 \pm 0.0012$. The uncertainties on the elements propagate forward in this calculation using the usual rules; we will suppress their value in the intermediate steps.

The true anomaly, f , at quadratures QI and QII as we have defined them is $f(\text{II}) = 180^\circ - 152.6^\circ = 27.4^\circ$; $f(\text{I}) = f(\text{II}) + 180^\circ = 207.4^\circ$. The eccentric anomaly, E , at QI and QII is given by

$$\tan(E/2) = [(1-e)/(1+e)]^{1/2} \tan(f/2) \quad [\text{C1}]$$

or $E(\text{II}) = 25.12^\circ$, $E(\text{I}) = 209.87^\circ$.

The distance between the centers of the two stars is given by

$$d = a[1 - e \cos E] \quad [\text{C2}]$$

Using $[a(x)/c] \sin i = 113.89 \pm 0.13$ s and $i = 76^\circ (+5^\circ, -9^\circ)$ (Dolan & Tapia 1988), we get $a(x) = 3.52 \times 10^{12}$ cm. Since $a = a(x) + a(o)$, if we estimate $M(o) = 10 M(x)$ then $a(x) = 10 a(o)$ and $a = 1.1 a(x) = 3.87 \times 10^{12}$ cm. Then from Eq. (C2), $d(\text{II}) = 3.55 \times 10^{12}$ cm and $d(\text{I}) = 4.17 \times 10^{12}$ cm.

The mean velocity of the neutron star in its apparent orbit is $\langle v \sin i \rangle = 2\pi a(x) \sin i / P = 276.96 \text{ km s}^{-1}$. From Kepler's Third Law, $v \propto r^{-1/2}$, where r is the distance from the neutron star to the COM. Then

$$v \sin i / \langle v \sin i \rangle = [\langle r \sin i \rangle / r \sin i]^{1/2} = [\langle r \rangle / r]^{1/2} \quad [\text{C3}]$$

Since $\langle r \rangle = a(x)$ and $r(\text{I, II}) \sin i = [a(x)/a]d(\text{I, II})$, we get $r(\text{II}) = 3.23 \times 10^{12}$ cm and $r(\text{I}) = 3.79 \times 10^{12}$ cm. Then from Eq. (C3), $v \sin i(\text{II}) / \langle v \sin i \rangle = \alpha = 1.043$; $v \sin i(\text{I}) / \langle v \sin i \rangle = \beta = 0.963$, and $\alpha/(\alpha - \beta) = 13.1 \pm 0.2$.

REFERENCES

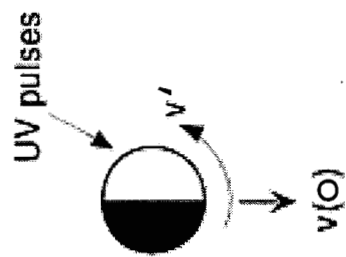
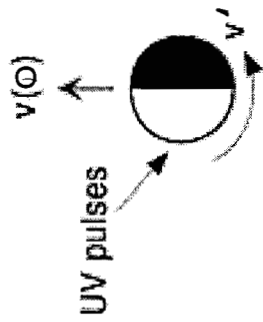
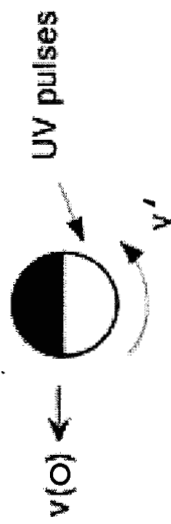
- Abubekеров, M. K., Antokhina, E. A., & Cherepashchuk, A. M. 2004, *Astron. Rep.*, 48, 89
- Alme, M. L., & Wilson, J. R. 1974, *ApJ*, 194, 147
- Arnett, W. D., & Bowers, R. L. 1977, *ApJ Suppl.*, 33, 415
- Avni, M. L., & Bahcall, J. N. 1974, *ApJ*, 191, 221
- Bahcall, J. 1978, *ARA&A*, 16, 241
- Barziv, O., Kaper, L., van Kerkwijk, M. H., Telting, J. H., & van Paradijs, J. 2001, *A&A*, 377, 925
- Basko, M. M., & Sunyaev, R. A. 1973, *Ap. Space Sci.*, 23, 117
- Bildsten, L., et al. 1997, *ApJ Suppl.*, 113, 367
- Boroson, B., McCray, R., Kallman, T., & Nagase, F. 1996, *ApJ*, 465, 940
- Boynton, P. E., Deeter, J. E., Lamb, F. K., & Zylstra, G. 1986, *ApJ*, 307, 545
- Brown, G. E., & Bethe, H. A. 1994, *ApJ*, 423, 659
- Clark, J. S., et al. 2002, *A&A*, 392, 909
- Dahab, R. E. 1974, *ApJ*, 187, 351
- Datta, B. 1988, *Fund. Cosmic Phys.*, 12, 151
- Deeter, J. E., Boynton, P. E., Lamb, F. K., & Zylstra, G. 1987, *ApJ*, 314, 634
- Dolan, J. F., et al. 1998, *A&A*, 331, 1037
- Dolan, J. F., & Tapia, S. 1988, *A&A*, 202, 124
- Dolan, J. F., et al. 1980, *ApJ*, 238, 238
- Glendenning, N. K. 1986, *Phys. Rev. Lett.*, 57, 1120

- Heap, S. R., & Corcoran, M. F. 1992, *ApJ*, 387, 340
- Hynes, R. I., Steeghs, D., Casares, J., Charles, P. A., & O'Brien, K. 2003, *ApJ*, 583, L95
- Inam, S. C., & Baykal, A. 2000, *A&A*, 353, 617
- Kaper, L., Hammerschlag-Hensberge, G., & Zuiderwijk, E. J. 1994, *A&A*, 289, 846
- Lattimer, J. M., & Prakash, M. 2004, *Science*, 304, 536
- Nice, D. J., Splaver, E. M., & Stairs, I. H. 2004, *Young Neutron Stars and Their Environments*, IAU Symp. 218 (F. Camilo & B. M. Gaensler, eds.)
- Schachter, J., Fillipenko, A. V., & Kahn, S. M. 1989, *ApJ*, 340, 1049
- Shapiro, S. L., & Teukosky, S. A. 1983, *Black Holes, White Dwarfs, and Neutron Stars* (New York, John Wiley), 241
- Steeghs, D., & Casares, J. 2002, *ApJ*, 568, 273
- Still, M. D., Quaintrell, H., Roche, P. D., & Reynolds, A. P. 1997, *MNRAS*, 292, 52
- Stock, R. 1989, *Nature*, 337, 319
- Thorsett, S. E., & Chakrabarty, D. 1999, *ApJ*, 512, 288
- Thorsett, S. E., Arzoumanian, Z., McKinnon, M. M., & Taylor, J. H. 1993, *ApJ*, 405, L29
- van Kerkwijk, M. H., van Paradijs, J., & Zuiderwijk, E. J. 1995a, *A&A*, 303, 497
- van Kerkwijk, M. H., et al. 1995b, *A&A*, 303, 483
- Zuiderwijk, E. J. 1995, *A&A*, 299, 79

FIGURE CAPTION

Fig. 1. The binary system at first quadrature (left) , second quadrature (right), and inferior conjunction of the X-ray source (top) as seen from the pole of the orbital plane. The direction to Earth is toward the bottom of the page. This illustration depicts a system with counterclockwise rotation on the sky and direct rotation of the primary, but the results are valid for any sense of rotation and non-zero inclination.

Inferior Conjunction



First Quadrature

Second Quadrature

To Earth



NS